Short-Term Planning of Cogeneration Power Plants: a Comparison Between MINLP and Piecewise-Linear MILP Formulations

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Abstract

In this work we compare two optimization approaches to tackle the short-term operational planning of energy systems including power plants, boilers, heat storage, as well as cogeneration units. We first describe a mixed-integer nonlinear programming formulation for the problem and then a mixed-integer linear one, obtained using piecewise-linear approximations of the nonlinear performance functions. We report and discuss numerical results on a set of realistic test cases, comparing the quality of the solutions and the computing time of the two approaches.

Keywords: cogeneration systems, mixed integer nonlinear optimization, piecewise approximation

1. Introduction

Nowadays quite complex energy systems are used to satisfy the electricity, heat and refrigeration power demand of industrial processes as well as buildings and cities (e.g., district heating networks). Such systems generally include not only conventional power plants, boilers, and refrigeration cycles, but also heat pumps, cogeneration units and heat storage systems. Among them, cogeneration units, also called Combined Heat and Power (CHP) plants, are particularly advantageous because of their improved integration of the heat flows which leads to remarkable savings in primary energy consumption and CO_2 emissions. On the other hand, the operational planning of these units is more challenging than that of conventional power plants as the two power outputs (electricity and heat) are interrelated. In addition, the presence of a heat storage unit further complicates the planning problem as it links all the time periods making time-decomposition-based techniques unsuitable. In short-term operational planning, given a set of cogeneration units and other possible generation and heat storage units, one has to determine for each time period of a time horizon which units must be switched on/off, the value of their operating variables, and the amount of stored energy in order to minimize an objective function, while satisfying the demands of electric and thermal power over all time periods. Since the performance functions of many cogeneration units are nonlinear due to the significant efficiency decrease at partial loads, the operational planning of a cogeneration system is a nonlinear mixed integer optimization problem.

In the literature, two main approaches are adopted to model energy and cogeneration systems. In *data-driven* approaches, the behavior of the energy systems is described with approximate models obtained from experimental data. One can consider an explicit approximation of the performance functions of the units, see, e.g., [7], for linear and nonlinear models. An alternative is to project out the input variables (typically fuel), and consider a convex-hull representation in the power-heat-cost space, see, e.g., [4] and [2]. In *first-principles* approaches, the system is decomposed into simpler components with well-known behavior, and thermodynamic balance equations are imposed to determine the plant operating points. This kind of approach is often necessary, for example, for complex CHP steam cycles and combined cycles with multiple operating variables, see, e.g., [3] and [6]. Both types of approach typically lead to mixed-integer (possibly nonlinear) optimization models, that can be tackled with mathematical programming techniques.

In this article we present a Mixed-Integer Nonlinear Programming (MINLP) formulation and a Mixed-Integer Linear (MILP) one, where the nonlinear performance functions are approximated using piecewise

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linear functions. In the selected computational results that we report, we compare the quality of the solutions and the efficiency of the two approaches with off-the-shelf exact MINLP and MILP solvers for some realistic instances of small-to-medium size and complexity.

2. The short-term operational planning problem

We consider cogeneration energy systems involving the following types of cogeneration units:
- One-degree-of-freedom cogenerative units that simultaneously generate electric and thermal power, e.g., gas turbines, internal combustion engines, fuel cells.
- Two-degree-of-freedom cogenerative units that simultaneously generate electric and thermal power (depending on two operating variables). This class includes, e.g., gas turbines with supplementary firing and steam cycles with extraction-condensing turbine.

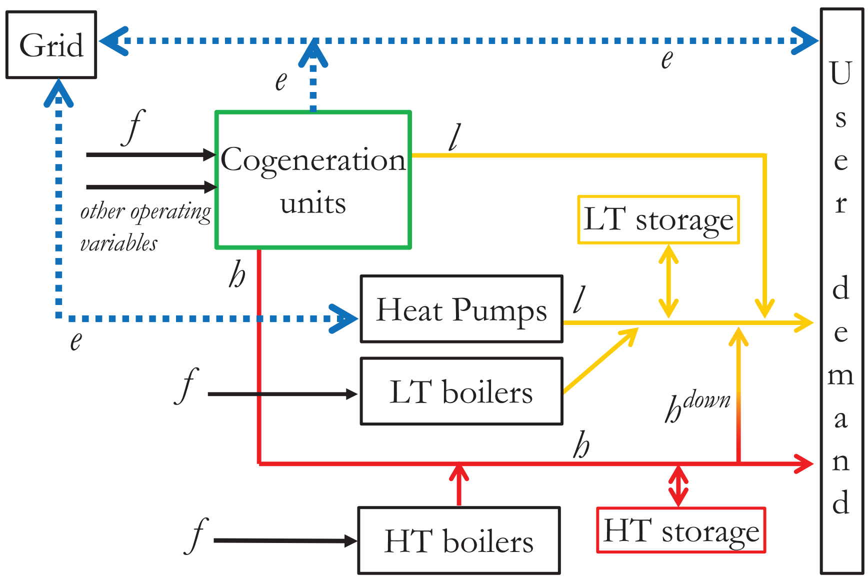

Figure 1: Schematic representation of a CHP system that cogenerates electricity and heat at two temperature levels.

The system may also include conventional generation units such as boilers and compression heat pumps. In addition, storage tanks may be connected to the heat network. The electric power generated by the units can be used to fulfill the customers' demands and, at the same time, drive the compression heat pumps. Electric power can also be sold/purchased to/from the electric grid.

Given a cogeneration system, including (co)generation units and heat storage tanks, time-dependent demands of low and high-temperature thermal and electric power, and time-dependent price of electricity, the short-term operational planning problem amounts to determining the schedule that minimizes the total operating costs, while satisfying the given demands for all the time periods in a given time horizon. Adopting a data-driven approach, we consider nonlinear performance functions derived from data, either experimental or provided by the manufacturer, that well approximate the behavior of each unit. We also account for the start-up phase of units, that may incur a significant energy penalty due to their warm-up phase, and there is an upper bound on the number of start-up operations.

3. MINLP formulation and MILP approximation

Using the sets, parameters and decision variables defined in the nomenclature, the short-term cogeneration systems planning problem can be formulated as the following MI(N)LP:

$$\begin{aligned}
\min \quad & \sum_{t\in\mathcal{T}} \left(\sum_{i\in\mathcal{U}} c_i^{\mathsf{OM}} z_{it} + \sum_{i\in\mathcal{U}} c_i^{\mathsf{SU}} \delta_{it} + \sum_{i\in\mathcal{F}} c_i^f f_{it} + b_t e_t^+ - p_t e_t^- \right) && && (1)\\
s.t. \quad & \sum_{i\in\mathcal{G}} e_{it}^{gen} - \sum_{i\in\mathcal{E}} e_{it}^{cons} + e_t^+ - e_t^- = D_t^e && \forall t\in\mathcal{T} && (2)\\
& \sum_{i\in\mathcal{H}} h_{it} - h_t^{down} + (u_t - \frac{u_{t+1}}{1-\alpha}) \geq D_t^{high} && \forall t\in\mathcal{T}, i\in\mathcal{U} && (3)\\
& \sum_{i\in\mathcal{L}} l_{it} + h_t^{down} + (v_t - \frac{v_{t+1}}{1-\beta}) \geq D_t^{low} && \forall t\in\mathcal{T}, i\in\mathcal{U} && (4)\\
& z_{it}F_{it}^{min} \leq f_{it} \leq z_{it}F_{it}^{max} && \forall t\in\mathcal{T}, i\in\mathcal{F} && (5)\\
& z_{it}E_{it}^{min} \leq e_{it}^{cons} \leq z_{it}E_{it}^{max} && \forall t\in\mathcal{T}, i\in\mathcal{E} && (6)\\
& \textit{performance constraints linking } z_{it}, f_{it}, e_{it}^{cons}, y_{it}, x_{it}, e_{it}^{gen}, h_{it}, l_{it} && \forall t\in\mathcal{T}, i\in\mathcal{U} && (7)\\
& \sum_{t\in\mathcal{T}} \delta_{it} \leq N_i && \forall t\in\mathcal{T}, i\in\mathcal{U} && (8)\\
& \delta_{it} \geq z_{it} - z_{it-1} && \forall t\in\mathcal{T}, i\in\mathcal{U} && (9)\\
& e_{it}^{gen}, h_{it}, l_{it}, h_t^{down}, e_t^+, e_t^- \geq 0 && \forall t\in\mathcal{T}, i\in\mathcal{U} && (10)\\
& 0 \leq u_t \leq U,\ 0 \leq v_t \leq V && \forall t\in\mathcal{T} && (11)\\
& 0 \leq \delta_{it} \leq 1,\ z_{it}\in\{0,1\} && \forall t\in\mathcal{T}, i\in\mathcal{U}. && (12)
\end{aligned}$$

The aim is to minimize the operational costs minus the revenue obtained by selling extra electricity to the grid. In the objective function (1), the start-up penalties c_i^{SU} account for the extra cost due to the warm-up phase, while the fixed cost c_i^{OM} accounts for operation and maintenance costs proportional to the number of working hours. Constraints (2) impose that the net amount of electric power must satisfy the demand D_t^e for period t. It is necessary to distinguish between the energy that is purchased from the power grid, e_t^+, from the one that is sold, e_t^-, since their price is different. Constraints (3) and (4), balance constraints for high and low-temperature heat, ensure that the thermal requirements in period t are satisfied. High-temperature heat can be downgraded to low-temperature via h^{down}. Thermal energy can be stored in the tank for the next period, as long as the capacities U, V are not saturated, and we account for constant loss rates $\alpha, \beta \in [0, 1)$. Thermal energy in excess can be dissipated with no additional costs. Constraints (5) and (6) ensure that the operating variables for a unit i (fuel, consumed electricity) are within the technical limits. Constraints (7), that model the nonlinear behavior of the generation units, are described in detail in the next two paragraphs. Constraints (8) and (9) limit the number of start-ups.

Nonlinear performance constraints. Each unit $i \in \mathcal{U}$ is described in terms of nonlinear performance functions $g_{it}(\cdot)$, that map one or more operating variables (fuel, consumed electricity, supplementary fuel) to an output variable (low or high-temperature heat, electric power). The performance functions are in general, non-convex and time-varying due to the non-negligible temperature effect. In addition, if unit $i \in \mathcal{U}$ is off, its output has to be 0. Thus, Constraints (7) can be expressed as inequalities of the form $\zeta \leq z_{it} g_{it}(\underline{\theta})$, where $\underline{\theta}$ is the vector of input variables and ζ an output variable, and the problem can be solved as a Mixed-Integer Nonlinear Program.

In the case of generation or cogeneration units with one degree of freedom (d.o.f.), each performance function g_{it} will be a function of one variable (θ is scalar). For instance, given a high-temperature auxiliary boiler, the output variable is high-temperature thermal power h_{it}, while the only operating variable is fuel f_{it}. The feasible region for h_{it} will be $\{0\} \cup [g_{it}(F_{it}^{min}), g_{it}(F_{it}^{max})]$. In the case of cogeneration units with more degrees of freedom, the performance functions depend on two or more operating variables ($\underline{\theta}$ is a vector). Two examples are combined cycles with extraction valve regulation (left) and gas turbines with post-firing (right):

$$\begin{cases} l_{it} \le z_{it} g^l_{it}(f_{it}, x_{it}) \\ h_{it} \le z_{it} g^h_{it}(f_{it}, x_{it}) \\ e^{gen}_{it} \le z_{it} g^e_{it}(f_{it}, x_{it}) \end{cases} \quad (13) \qquad \begin{cases} l_{it} \le z_{it} g^l_{it}(f_{it}, y_{it}) \\ h_{it} \le z_{it} g^h_{it}(f_{it}, y_{it}) \\ e^{gen}_{it} \le z_{it} g^e_{it}(f_{it}, y_{it}), \end{cases} \quad (14)$$

where the operating variables are the fuel quantity f_{it}, the valve opening percentage $x_{it} \in [0, 0.4]$ for the combined cycle (13) and the supplementary fuel y_{it} for the gas turbine (14).

Piecewise linear approximation. An alternative approach consists in approximating the nonlinear performance functions with piecewise linear functions, see e.g. [1], obtaining a more tractable Mixed-Integer Linear Program (MILP). The piecewise linear approximation of 1-d.o.f. performance functions is rather straightforward, as it is sufficient to select a set of discretization points on a line, and connect them via line segments. For 2-d.o.f. units the approximation involves functions of two variables. Several approaches are available for approximating 2-D functions, differing considerably in terms of accuracy of the approximation and computational cost of the resulting MILP. In our model, we consider the so-called *lambda method* described, e.g., in [5], that is implemented by triangulating the domain of the nonlinear function. Then, the value in a point $\underline{x}$ is computed as the convex combination of the function values in the vertices of the triangle containing $\underline{x}$. This method requires the introduction of $O(n_1 \times n_2)$ binary variables, where n_1 and n_2 are the number of discretization points per dimension.

4. Computational experiments

Given the wide variety of cogeneration systems, ranging from small to large scale, we consider two scenarios from which we build several test instances.

Scenario 1. The first scenario is a micro-cogeneration system designed to provide low and high-temperature thermal power and electricity to a large building. The system consists of:

- a Solid Oxide Fuel Cell (SOFC) using natural gas to cogenerate electric and thermal power;
- a Heat Pump (HP) using electric power to generate low temperature heat;
- an Auxiliary Boiler (AB), mainly used as a backup;

and a thermal storage for high-temperature heat energy. For this scenario, we consider a single instance with 3 units that we indicate with `1-a`.

Scenario 2. The second scenario is a large scale cogeneration system providing heat to a district heating network. The system may include one or more of the following units:

- Gas Turbines (GT) with heat recovery;
- Gas Turbines (GT-2) with supplementary firing and heat recovery;
- Natural Gas Combined Cycles (NGCC) with a bottoming back-pressure steam turbine;
- Natural Gas Combined Cycles (NGCC-2) with a bottoming extraction-type steam turbine;
- Auxiliary Boilers (AB) burning natural gas to generate heat;

and a thermal storage for high-temperature heat energy. Since only thermal power is required, the whole amount of electricity cogenerated by gas turbines and combined cycles is sold to the grid. This scenario includes units with two degrees of freedom, see Eq. (13) and (14): in GT-2, supplementary fuel can be burned to increase the amount of recovered heat, and, in NGCC-2, opening the steam extraction valve reduces the electric power and increases the thermal power. Four different instances are considered (`2-a`, `2-b`, `2-c` and `2-d`), with, respectively, 5, 4, 12 and 11 (co)generation units. Instances `2-b` and `2-d` include also 2-d.o.f. cogeneration units.

Computational experiments were performed, for the MINLP formulations, with the open-source solver SCIP 3.1.0, while for the MILP formulations IBM Ilog CPLEX 12.6 was used. For the MINLP, we have also experimented with BARON, whose results are not included for sake of brevity, since its efficiency on the considered instances was slightly inferior. The tests were carried out on an Intel Xeon with E3125@3.30GHz CPUs and 16GB of RAM, with a time limit of 2 hours.

Table 1 summarizes the computational results. The MINLP instances turn out to be challenging. SCIP is able to certify optimality for 2 out of 5, and on `2-c` is close to the optimum, while the instances with 2-d.o.f. units are harder. In comparison, the MILP formulations can be solved to optimality by CPLEX in a few

Table 1: Optimal values, computing time (seconds) and lower/upper bounds for the MINLP and the approximate MILP with an increasing number of discretization points (d.p.) per dimension.

	2 d.p.		3 d.p.		5 d.p.		9 d.p.		15 d.p.		MINLP			
	time	opt	time	opt	time	opt	time	opt	time	opt	time	LB	UB	gap
`1-a`	0.01	94.33	0.03	91.73	0.06	91.30	0.10	91.17	0.42	91.11	42.58	91.07	**91.07**	0.0
`2-a`	0.04	104.17	0.07	102.49	0.10	101.95	0.12	101.86	0.14	101.82	812.8	101.76	**101.76**	0.0
`2-b`	0.06	-80.12	0.14	-80.24	1.19	-80.33	4.53	-80.36	34.44	-80.36	7200	-94.36	-71.97	31.1
`2-c`	0.15	307.01	0.04	302.73	0.09	300.75	0.09	300.51	0.17	300.43	7200	284.83	300.19	5.4
`2-d`	0.22	121.27	0.29	119.43	0.31	118.20	0.50	118.09	3.43	118.07	7200	-83.01	140.93	∞

seconds. The MILP solutions are not necessarily feasible for the original formulation, since the approximate model might overestimate the amount that is generated. However, it is always possible to recover infeasibility *a posteriori* by increasing the production level. Interestingly, on our instances all the MILP optimal solutions are feasible (barring minor numerical errors), as they tend to be on the discretization points.

The results show that the approximate optimal values approach the optimal value of the original formulation as the number of discretization points is increased. The optimal values are significantly different when the approximation is less accurate – except for instance `2-b`, where the variation is small, since the solution is dominated by a large NGCC-2 unit always at full load.

The structure of the solutions can differ significantly. As an example, we report in Figure 2 two optimal schedules for a low-temperature heat pump in instance `1-a` with a 2-point MILP approximation (left) and with the MINLP model (right). Although the instance is simple, the structure of the optimal approximate solution differs from that of the optimal MINLP solution even when the number of discretization points is increased to 5. To obtain optimal solutions to these two problems that are equivalent, one needs to use at least 9 discretization points.

5. Concluding remarks

The summarized computational results for two relatively simple scenarios of the short-term operational planning problem indicate that even small-size instances of the MINLP can be computationally very challenging. Approximating the nonlinear performance functions with piecewise linear functions is an alternative that seems to work quite well in practice. For the considered instances, the resulting approximate MILP models can generally be solved more efficiently than their MINLP counterparts, and they appear to be already fairly accurate with a few linear pieces.

Attention must be paid to the feasibility of the solutions obtained with the approximations. Indeed, if the optimal operating point of a unit is far from the approximation discretization points, the piecewise linear function value might be quite different from the actual value. Underestimating the actual performance value may lead to suboptimal solutions with more than 3% error, while overestimating it may lead to infeasible solutions.

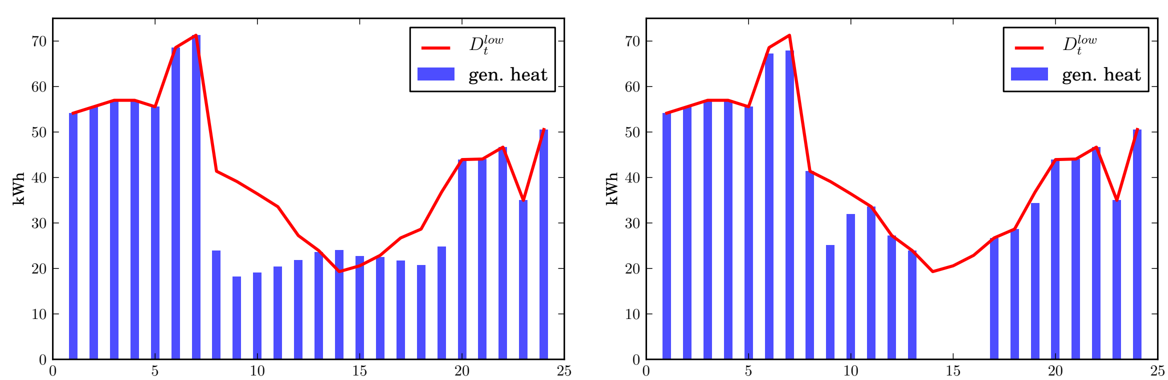

Figure 2: Optimal plan for the heat pump of instance `1-a` obtained with a 2-point piecewise approximation (left) and with the MINLP (right).

Nomenclature

$\mathcal{T}$: set of time periods (hours)
$\mathcal{U}$: set of all generation units
$\mathcal{F}$: set of units consuming fuel
$\mathcal{E}$: set of units consuming electricity
$\mathcal{H}$: set of units that generate high-temperature heat
$\mathcal{L}$: set of units that generate low-temperature heat
$\mathcal{G}$: set of units that generate electricity
c_i^{OM}: hourly operation and maint. cost for unit i [€]
c_i^{SU}: start-up cost for unit i [€]
c_i^f: unit cost of fuel for unit i [€/kWh]
b_t, p_t: unit price of electricity bought/sold from/to the grid at time t [€/kWh]
$F_{it}^{min}, F_{it}^{max}$: minimum and maximum fuel input for unit $i \in \mathcal{F}$ at time t [kWh]
$E_{it}^{min}, E_{it}^{max}$: minimum and maximum electricity input for unit $i \in \mathcal{E}$ [kWh]
N_i: maximum number of start-ups for unit i
U, V: capacity of low and high-temperature heat storage [kWh]
α, β: constant loss rate for thermal storage [%]
$D_t^{low}, D_t^{high}, D_t^e$: demand for low and high-temperature heat, electricity at time t [kWh]
f_{it}: fuel consumed by unit $i \in \mathcal{F}$ in period t [kWh]
y_{it}: secondary fuel consumed by unit $i \in \mathcal{F}$ with post-firing injection [kWh]
x_{it}: extraction valve opening percentage for combined cycle units [%]
e_{it}^{cons}: electricity consumed by $i \in \mathcal{E}$ in period t [kWh]
e_{it}^{gen}: electricity generated by $i \in \mathcal{G}$ in period t [kWh]
l_{it}: low-temperature heat generated by unit $i \in \mathcal{L}$ in period t [kWh]
h_{it}: high-temperature heat generated by unit $i \in \mathcal{H}$ in period t [kWh]
h_t^{down}: high-temperature heat downgraded to low-temperature in period t [kWh]
e_t^+, e_t^-: electricity bought/sold from/to the grid in period t [kWh]
u_t, v_t: high and low-temperature thermal energy stored at the beginning of period t [kWh]
z_{it}: binary variable, on/off status of unit i in period t
δ_{it}: binary start-up variable ($\delta_{it} = 1$ if unit i is switched on at beginning of period t)
$g_{it}^h, g_{it}^l, g_{it}^e$: performance functions for unit i at time t

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