Short-term planning of cogeneration energy systems via Mixed-Integer Nonlinear Optimization

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1 Introduction

Combined Heat and Power (CHP) plants, also called *cogeneration power plants*, are energy systems composed of a network of units that convert primary energy (fossil fuels) into electricity and useful heat so as to meet the demand of electric power and heat at certain temperature levels of a set of users. As in a cascade process, primary energy is converted into electric power through a thermodynamic cycle, and the heat discharged by the cycle is used to satisfy the users' heat demand. Thanks to the improved integration of these heat flows, CHP plants achieve remarkable savings in primary energy and in CO_2 emissions with respect to non-cogenerative plants at both large [7] and small scales [13]. Therefore, several European and North American countries have recently adopted incentive policies to strongly favor CHP plants as well as Combined Cooling Heat and Power (CCHP) plants that also cogenerate refrigeration power.

Due to its practical relevance, the optimization of cogeneration systems has received a growing attention during the last decade. The problems addressed range from design optimization and long-term tactical planning of energy plants, to short-term operational planning, where the components of the energy systems are considered in greater detail.

In short-term operation planning, given a set of cogeneration units and other possible generation and heat storage units, one has to determine for each time period t of a given time horizon which units must be switched on/off, the value of their operating variables (e.g., input fuel), and the amount of stored energy in order to minimize an objective function (e.g., the total operating costs), while satisfying the demands of electric, thermal and refrigeration power. In addition, electrical energy can be sold/purchased to/from the power grid, and the price of electrical energy can vary hourly in deregulated markets. Since the different cogeneration units (e.g., multiple CHP gas turbines) can be independently controlled (e.g., switched on/off) and the performance curves of several cogeneration units are nonlinear due to the significant efficiency decrease at partial loads, the short-term operational planning of a cogeneration system is a nonlinear mixed integer optimization problem.

In this chapter we describe a basic version of the problem, present a Mixed-Integer Nonlinear Programming (MINLP) formulation and summarize some computational results obtained with state-of-the-art MINLP global solvers, namely BARON [21], SCIP [1] and Couenne [3], on some realistic instances of small-to-medium size and complexity. As we shall see, even small-size and relatively simple instances of the short-term cogeneration systems planning problem can be very challenging.

2 Related work

In the literature, two main approaches are adopted to tackle short-term operational planning of cogeneration systems: a data-driven one and one based on first-principles, which differ in the way the behaviour of the (co)generation units is modelled. In both cases, combinatorial constraints can be included to model the commitment of the units and related constraints on ramp rates, start-up/shut-down costs, etc..

In data-driven *black-box* approaches, the behavior of the energy systems is described with approximate models obtained from experimental data. This is a rather common approach that gives considerable flexibility with respect to the level of accuracy in the description of the system. It is possible to consider an explicit approximation of the performance curves of the units in the system, see, e.g., [4] and [23] that consider linear and nonlinear models, approximated via piecewise linear functions. An alternative is to consider only the space of the output variables (heat and electric power), projecting out the input variables (fuel, consumed electricity), either considering linear costs [2], or a convex-hull representation in the power-heat-cost space, as done in [14] and [8]. These representations typically lead to linear programming models, or mixed-integer linear programming ones if discrete unit operation modes and other non-convexities are accounted for [15].

In first-principles thermodynamic approaches, the system is decomposed into simpler components with well-known performance curves, and mass/energy balance equations are imposed to determine the plant operating points. This kind of approach can be adopted, for example, for complex CHP steam cycles and combined cycles with multiple operating variables and highly complex behaviors. Examples are [9] and [16], where the behavior of each component is described starting from specific thermodynamic relations.

The cogeneration system operational planning problem can be considered as a variant of what is known in the power systems community as the Unit Commitment (UC) problem [17]. UC consists of determining when to start up and shut down the power plants, and how much each committed unit should generate to meet the demand, while typically minimizing a quadratic cost function. Successful approaches for UC include dynamic programming for simple cases and Langrangian methods for more complex, large-scale problems (e.g., [11, 19]). In recent years, growing attention has been devoted to mathematical programming approaches (e.g., [6, 10]) due to substantial advances in Mixed Integer Programming theory and practice.

The core structure of the problem we address here, i.e., the production planning with presence of storage, also shares several similarities with the multi-item, multi-machine lot-sizing problem with bounded inventory and minimum lot size. For an extensive account on lot-sizing problems and in particular on polyhedral approaches, see e.g. [18] and the references therein.

3 Cogeneration energy systems

Depending on the actual application and setting, real-world cogeneration systems can substantially vary in terms of number and types of units as well as in terms of scale, ranging from small scale plants (applications with ¡ 50 kW fuel input) to large scale ones (industrial applications with ¿100 MW fuel input). Here, we consider cogeneration energy systems involving the following types of cogeneration units:

- One-degree-of-freedom cogeneration units that simultaneously generate electric and thermal power, e.g., gas turbines, internal combustion engines, back-pressure steam cycles, fuel cells.

- Two-degree-of-freedom cogeneration units that simultaneously generate electric and thermal power (depending on two operating variables). This class includes, for instance, gas turbines with supplementary firing in the heat recovery section, steam cycles with extraction-condensing turbine, combined cycles with supplementary firing in the Heat Recovery Steam Generator (HRSG) and back-pressure bottoming cycle.

Cogeneration systems may also include generation units such as:

- boilers (i.e., one-degree-of-freedom units generating only heat from fuel),

- compression heat pumps (i.e., one-degree-of-freedom units generating only heat from electricity),

- compression chillers (i.e., one-degree-of-freedom units generating only refrigeration power from electricity),

- absorption chillers (i.e., one-degree-of-freedom units generating only refrigeration power from heat).

The above-mentioned types of units allow to account for a wide variety of cogeneration systems involving units with multiple degrees of freedom (two or more operating variables) and different size.

Figure 1 gives a schematic representation of a cogeneration system comprising multiple cogeneration and generation units as well as networks for the distribution of electric power, refrigeration power, high and low temperature thermal power. For instance, the HT heat network allows to model a steam network for an industrial heat user, while the LT heat network accounts for a district heating network. Storage tanks can be connected to the heat networks as well as to the refrigeration power network. The electric power generated by the units can be used to fulfill the customers' demands and, at the same time, drive the compression heat

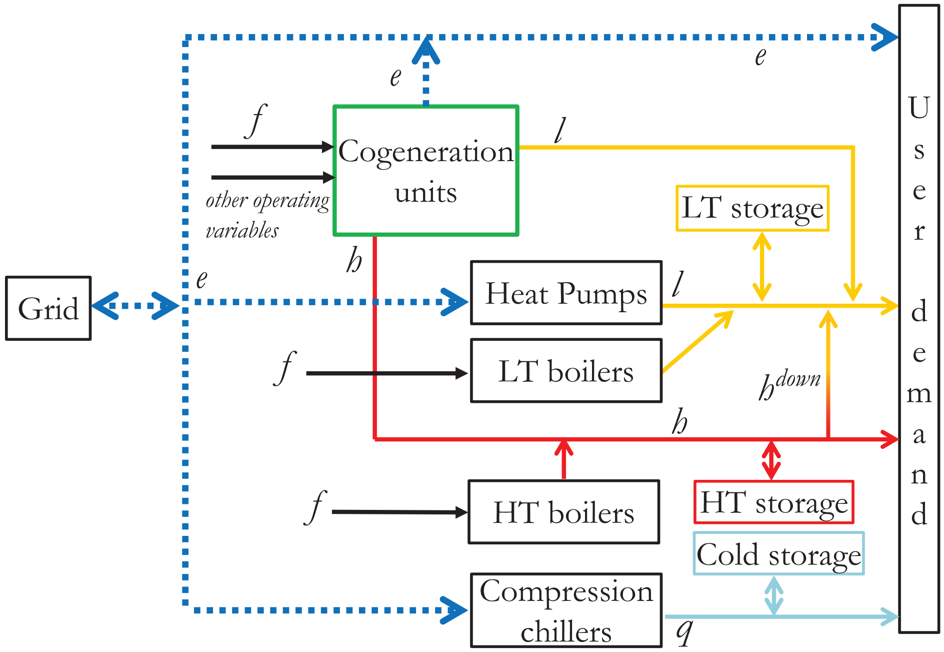


Figure 1: Schematic representation of a CCHP network connecting the (co)generation units with the storage tanks, the electric grid and the users. Red and yellow arrows represent, respectively, the high (h) and low-temperature (l) thermal power flows, light blue arrows represent the refrigeration power flows (q), blue dotted arrows represent the electric power (e), and the black ones the fuel (f) consumed by each unit.

pumps and compression chillers, and satisfy the electricity needs of the absorption chillers. Electric power can be sold/purchased to/from the electric grid. The HT and LT heat networks are interconnected in order to have the possibility to downgrade high-temperature heat down to the low temperature heat network. Finally, thermal power in excess can, if needed, be dissipated through a dedicated heat exchanger.

4 The basic problem and its peculiarities

The basic version of the short-term cogeneration system planning problem we consider is defined as follows. Given

- a cogeneration system as described above, including CCHP cogeneration units with possibly other generation units and heat storage tanks with fixed capacity,
- time-dependent demands of low and high-temperature thermal and electric power,
- time-dependent price of electricity,
- time-dependent ambient temperatures,

determine, for each time period $t \in \mathcal{T}$, the schedule that minimizes the total operating costs while satisfying the given demands.

We adopt a data-driven approach and consider nonlinear performance curves derived from data, either obtained experimentally or provided by the manufacturer, that approximate well the behavior of each unit. The unit performance curves can be time-varying, as the ambient temperature affects performance. Units can cogenerate electric power, thermal power and refrigeration power starting from fuel, electricity or heat. We also account for the start-up phase of some units, that may incur in a significant energy penalty due to their warm-up phase. Additional logical constraints can also be included to limit the number of start-up operations.

Let us now briefly summarize the main peculiarities of the short-term cogeneration system planning problem.

Compared to classical UC problems, a CCHP system includes not only the generation of electric power, but also of other commodities, such as thermal power (at different temperature levels) and refrigeration power. Note also that cogeneration units produce both electric power and thermal power simultaneously, and the cost cannot be considered simply as a quadratic function of the production level. Unlike in the usual plant-level approach of UC, single components of the cogeneration systems are considered in greater detail. Their interdependence is also crucial, because commodities can be converted (with some caveats): high-temperature heat can be easily converted to low-temperature heat (not the opposite), electricity can be used to generate thermal power via a heat pump, and so on. A crucial feature of our problem is also the possibility of storing thermal energy from one time period to the following one. The storage can be accessed by multiple units, effectively making a decomposition harder.

Compared to the classical multi-item, multi-machine lot-sizing problem with bounded inventory, our operational planning problem involves a non-convex objective function, complex interdependence between the 'items', and does not include an explicit inventory cost. Moreover, stored energy is subject to losses, as stored thermal energy decreases naturally with time.

5 MINLP formulation

Sets and parameters

$\mathcal{T}$: set of time periods (hours)

$\mathcal{U}$: set of all generation units

$\mathcal{F}$: set of units consuming fuel

$\mathcal{E}$: set of units consuming electricity

$\mathcal{C}$: set of units that generate refrigeration

$\mathcal{H}$: set of units that generate high-temperature heat

$\mathcal{L}$: set of units that generate low-temperature heat

$\mathcal{G}$: set of units that generate electricity

c_i^{OM}: hourly operation and maintenance cost for unit $i \in \mathcal{U}$ [€]

c_i^{δ}: start-up cost for unit $i \in \mathcal{U}$ [€]

c_i^{f}: unit cost of fuel consumed by unit $i \in \mathcal{F}$ [€/kWh]

b_t: unit price of electricity bought from the grid at time t [€/kWh]

p_t: unit price of electricity sold to the grid at time t [€/kWh]

$F_{it}^{min}, F_{it}^{max}$: minimum and maximum fuel input for unit $i \in \mathcal{F}$ at time t [kWh]

$E_{it}^{min}, E_{it}^{max}$: minimum and maximum electricity input for unit $i \in \mathcal{E}$ [kWh]

N_i: maximum number of start-ups for unit $i \in \mathcal{U}$

U, V, W: capacity of low/high-temperature heat and refrigeration storage [kWh]

α, β, γ: constant deterioration rate for thermal and refrigeration storage

$D_t^{low}, D_t^{high}, D_t^{cold}, D_t^{e}$: demand for low and high-temperature heat, refrigeration power, electricity at time t [kWh]

Decision variables

f_{it}: fuel consumed by unit $i \in \mathcal{F}$ in period t [kWh]

y_{it}: secondary fuel consumed by unit $i \in \mathcal{F}$ with post-firing injection [kWh]

x_{it}: extraction valve opening percentage for combined cycle units [%]

e_{it}^{cons}: electricity consumed by unit $i \in \mathcal{E}$ in period t [kWh]

e_{it}^{gen}: electricity generated by unit $i \in \mathcal{G}$ in period t [kWh]

l_{it}: low-temperature heat generated by unit $i \in \mathcal{L}$ in period t [kWh]

h_{it}: high-temperature heat generated by unit $i \in \mathcal{H}$ in period t [kWh]

h_t^{down}: high-temperature heat downgraded to low-temperature in period t [kWh]

q_{it}: refrigeration energy generated by unit $i \in \mathcal{C}$ in period t [kWh]

e_t^-: electricity sold to the grid in period t [kWh]

e_t^+: electricity bought from the grid in period t [kWh]

u_t: high-temperature thermal energy stored at the beginning of period t [kWh]

v_t: low-temperature thermal energy stored at the beginning of period t [kWh]

w_t: refrigeration energy stored at the beginning of period t [kWh]

z_{it}: binary variable, on/off status of unit i in period t

δ_{it}: binary start-up variable ($\delta_{it} = 1$ if unit i is switched on at beginning of period t)

Using the above-mentioned sets, parameters and decision variables the basic version of the short-term cogeneration systems planning problem can be formulated as the following MINLP:

$$\min \quad \sum_{t \in \mathcal{T}} \left(\sum_{i \in \mathcal{U}} c_i^{\mathsf{OM}} z_{it} + \sum_{i \in \mathcal{U}} c_i^{\delta} \delta_{it} + \sum_{i \in \mathcal{F}} c_i^f f_{it} + b_t e_t^+ - p_t e_t^- \right) \tag{1}$$

$$\begin{aligned}
s.t. \quad & \sum_{i \in \mathcal{G}} e_{it}^{gen} - \sum_{i \in \mathcal{E}} e_{it}^{cons} + e_t^+ - e_t^- = D_t^e & \forall t \in \mathcal{T} \quad & (2)\\
& \sum_{i \in \mathcal{H}} h_{it} - h_t^{down} + (u_t - \frac{u_{t+1}}{1-\alpha}) \geq D_t^{high} & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (3)\\
& \sum_{i \in \mathcal{L}} l_{it} + h_t^{down} + (v_t - \frac{v_{t+1}}{1-\beta}) \geq D_t^{low} & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (4)\\
& \sum_{i \in \mathcal{C}} q_{it} + (w_t - \frac{w_{t+1}}{1-\gamma}) \geq D_t^{cold} & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (5)\\
& z_{it} F_{it}^{min} \leq f_{it} \leq z_{it} F_{it}^{max} & \forall t \in \mathcal{T}, i \in \mathcal{F} \quad & (6)\\
& z_{it} E_{it}^{min} \leq e_{it}^{cons} \leq z_{it} E_{it}^{max} & \forall t \in \mathcal{T}, i \in \mathcal{E} \quad & (7)\\
& \textit{Performance constraints described in } (13) & & \\
& \sum_{t \in \mathcal{T}} \delta_{it} \leq N_i & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (8)\\
& \delta_{it} \geq z_{it} - z_{it-1} & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (9)\\
& e_{it}^{gen}, h_{it}, l_{it}, q_{it}, h_t^{down}, e_t^+, e_t^- \geq 0 & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (10)\\
& 0 \leq u_t \leq U,\ 0 \leq v_t \leq V,\ 0 \leq w_t \leq W & \forall t \in \mathcal{T} \quad & (11)\\
& \delta_{it} \in \{0,1\}, z_{it} \in \{0,1\} & \forall t \in \mathcal{T}, i \in \mathcal{U} \quad & (12)
\end{aligned}$$

The aim is to minimize the operational costs minus the revenue obtained by selling extra electricity to the grid. In the objective function (1), we consider unit-dependent fuel costs c_i^f. Start-up penalties c_i^δ account for the extra cost due to the the warm-up phase. The fixed cost c_i^{OM} accounts for Operation and Maintenance costs proportional to the number of working hours. It can include the cost of staff needed to operate and maintain the unit, or machine deterioration costs.

Constraints (2) are balance equations for electricity. The net amount of electric power, either generated or bought, must satisfy the demand D_t^e for period t. Note that some units generate electric power, while others consume electricity. It is necessary to separate energy that is purchased from the power grid, e_t^+, from the one that is sold, e_t^-, since their price is different. Constraints (3) are balance constraints for high-temperature heat. The requirement D_t^{high} for period t must be covered by the generated high-temperature heat and/or by that which is available in the storage (u_t). High-temperature heat can be downgraded to low-temperature. Thermal energy can be stored in the tank for the next period, as long as the capacity U is not saturated. Accordingly, the stored energy at the beginning of the following period will be:

$$u_{t+1} = \min\left\{U,\ (1-\alpha)\left(\sum_{i\in\mathcal{H}} h_{it} - h_t^{down} + u_t - D_t^{high}\right)\right\},$$

where $\alpha \in [0,1)$ is the constant deterioration rate for high-temperature heat. We assume that thermal energy in excess can be dissipated with no additional costs. Similarly, Constraints (4) are balance constraints for low-temperature heat, where we also include the high-temperature heat that has been downgraded to low-temperature one. Constraints (5) are balance constraints for the refrigeration units. Costraints (6) and (7) ensure that the operating variables for a unit i (fuel, consumed electricity) are within the technical minimum and maximum. Constraints (13), that model the nonlinear behaviour of the generation units, linking operating and output variables, are described in detail in the next paragraph. Constraints (8) and (9) limit the number of startups in the time horizon. Finally, constraints (10)-(12) impose lower and upper bounds, and integrality for variables z_{it}.

Nonlinear performance constraints The performance of each unit $i \in \mathcal{U}$ is described in terms of a nonlinear function $g_{it}(\cdot)$ for each period t, that maps one or more operating variables (fuel, consumed electricity, supplementary fuel) to an output variable (low or high-temperature heat, refrigeration power, electric power). The performance curves, which are usually continuous and nondecreasing, are often non-convex (sometimes even non-differentiable) and time-varying due to the non-negligible temperature effect in each period t. In addition, if unit i is off, its output has to be 0. Thus, the corresponding output variables are semi-continuous. The performance constraints for the generation units can then be expressed as inequalities of the form:

$$0 \le \zeta \le z_{it} g_{it}(\underline{\theta}) \tag{13}$$

where $\underline{\theta}$ is the vector of input variables, and ζ an output variable.

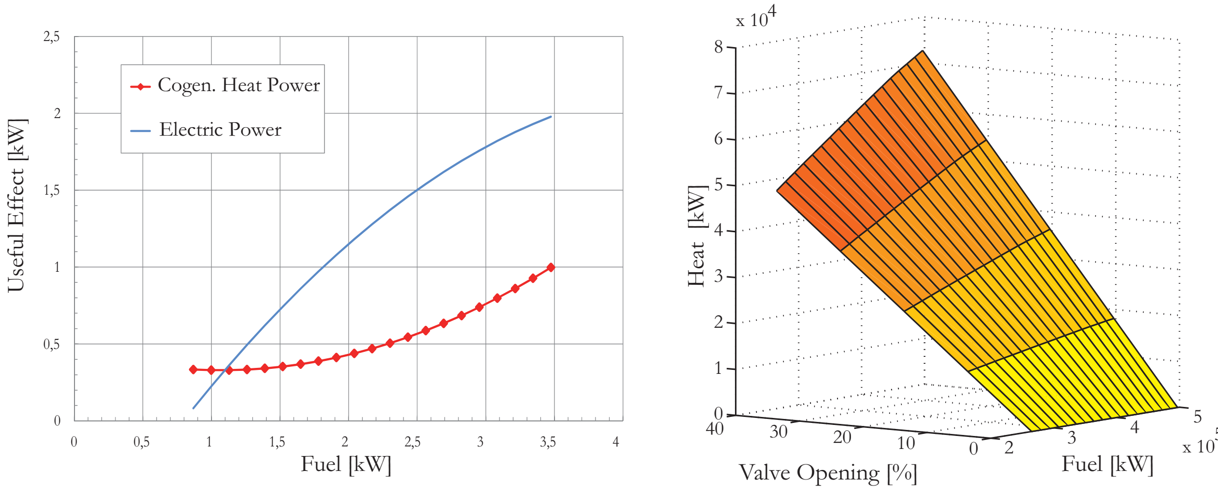
In the case of generation or cogeneration units with one degree of freedom, each performance curve g_{it} is a function of one variable (θ is scalar). For instance, given a high-temperature auxiliary boiler, the output variable is high-temperature thermal power h_{it}, while the only operating variable is fuel f_{it}. The feasible region for h_{it} will be $\{0\} \cup [g_{it}(F_{it}^{min}), g_{it}(F_{it}^{max})]$.

In the case of cogeneration units with more degrees of freedom, the performance curves are functions of two or more operating variables ($\underline{\theta}$ is a vector). Two examples are combined cycles with extraction valve regulation (left) and gas turbines with post-firing (right):

$$\begin{cases} l_{it} \le z_{it} g_{it}^l(f_{it}, x_{it}) \\ h_{it} \le z_{it} g_{it}^h(f_{it}, x_{it}) \\ e_{it}^{gen} \le z_{it} g_{it}^e(f_{it}, x_{it}) \end{cases} \tag{14}$$

$$\begin{cases} l_{it} \le z_{it} g_{it}^l(f_{it}, y_{it}) \\ h_{it} \le z_{it} g_{it}^h(f_{it}, y_{it}) \\ e_{it}^{gen} \le z_{it} g_{it}^e(f_{it}, y_{it}) \end{cases} \tag{15}$$

where the operating variables are the fuel quantity f_{it}, the valve opening percentage $x_{it} \in [0, 0.4]$ for the combined cycle (14) and the supplementary fuel y_{it} for the gas turbine (15). The variable y_{it} has a positive cost that must be added to the objective function, and must satisfy an additional technical constraint $y_{it} \le a + df_{it}$ with a, $d \ge 0$.



(a) (b)

Figure 2: (a) Useful effect (electric and thermal power) of a fuel cell unit as a function of consumed fuel. Note that the concavity is different: at larger loads the thermal efficiency increases, while the electrical efficiency decreases. The performance curves are derived from the data of a commercially available machine. (b) Heat as a function of fuel and extraction valve opening percentage for a natural gas combined cycle. Heat production is 0 when the valve is closed, and it increases with fuel when the valve is opened. Data obtained via simulation with the dedicated software Thermoflex [22].

Note that Constraints (13) are rather general. They can be adapted to several types of generation and cogeneration units, as long as it is possible to model their behavior as nonlinear functions of one or more operating variables.

As to the properties of the performance curves, it is worth pointing out that the units cannot always be classified a priori according to their convexity/concavity because it depends not only on the type of unit but also on the control strategy implemented by the manufacturer. For example, while boilers and heat pumps typically have concave performance curves, the performance curve of gas turbines is often neither concave nor convex, and even non-smooth, due to a change of control strategy occurring at about 60% of the load.

Extensions The model can be extended with additional features and constraints. For example, it is possible to introduce other temperature levels for thermal power. In some cases, it is desirable to model the transition states of the generation units more accurately with ramp-up and ramp-down constraints. Technical constraints regarding temperature limits, mutually exclusive units, minimum and maximum up/down-time are also common in similar problems. Moreover, in large-scale systems, the topological aspect of the distribution network, for both heat and power, could be taken into account.

6 Computational experiments

Given the wide variety of cogeneration systems, ranging from small to large scale, we consider two scenarios: a domestic application (first scenario) with a few small-size cogeneration units, and an industrial application (second scenario) with a considerable number of larger-size cogeneration units. Six instances generated from the scenarios are available from [20].

6.1 Scenario 1

The first scenario is a micro-cogeneration system designed to provide thermal power, refrigeration power and electricity to a $2,000\ m^2$ building. More in detail, the building has the following power requirements: high-temperature thermal power (hot water above 60 °C) for domestic hot water, low temperature thermal power (hot water $35 - 45$ °C) for heating, refrigeration power for air conditioning during summer period, electric power. The cogeneration system is made of the following units:

- a Solid Oxide Fuel Cell (SOFC) using natural gas to cogenerate up to 30kW and 15kW of, respectively, electric and thermal power;
- a Heat Pump (HP) using electric power to generate low temperature heat by "pumping" heat from ambient temperature up to 35-45 °C. It generates about 130kW at nominal conditions, but it is very sensitive to ambient temperature.
- an Auxiliary Boiler (AB) burning natural gas to generate up to 100kW of high-temperature heat;
- a thermal storage system to store up to 100kWh of heat energy.

Figure 2a shows the performance curves of the SOFC units, i.e., the useful effects, heat and electric power as a function of the fuel input. Due to the fact that the thermal and electric request may have independent time profiles, the heat storage tank is essential in order to allow the cogeneration system to generate extra electric power (to be sold to the grid) when the selling price is higher without wasting the cogenerated heat (which will be stored and used when needed). The auxiliary boiler is included in the system mainly as a backup and it is capable to fulfill the requirement peaks of both high and low temperature heat.

6.2 Scenario 2

The second scenario is a large scale cogeneration system providing heat to a district heating network. The requirement is thermal power at one level of temperature, about 90 °C, while the whole electricity production is sold to the electric grid. The cogeneration system includes one or more of the following units[1]:

- Gas Turbines (GT) with heat recovery, burning natural gas to generate up to about 10MW of heat and 5.5MW of electricity;
- Gas Turbines (GT-2) with supplementary firing and heat recovery, burning natural gas to generate up to about 40MW of heat and 11MW of electricity;
- Natural Gas Combined Cycles (NGCC) with a bottoming back-pressure steam turbine, burning natural gas to generate up to 30MW of heat and 45MW of electricity;
- Natural Gas Combined Cycles (NGCC-2) with a bottoming extraction-type steam turbine, burning natural gas to generate up to about 70MW of heat and 30MW of electricity;
- Auxiliary Boilers (AB) burning natural gas to generate up to about 40MW of heat;
- a thermal storage system to store up to 50MWh of heat energy.

The thermal power requirements are fulfilled by well established CHP units, like gas turbines and combined cycles, with the help of auxiliary boilers. This scenario includes cogeneration units with two degrees of freedom, namely, gas turbines (GT-2) with post-firing injection, and combined cycles with extraction condensing steam turbine (NGCC-2). In GT-2, it is possible to burn supplementary fuel to increase the amount of heat that can be recovered from the exhaust gases (15). In NGCC-2, the amount of cogenerated heat and electric power is a function of the consumed fuel and the opening of a steam extraction valve, as described in (14). Opening the valve reduces the electric power efficiency and increases the amount of recovered heat, while closing the valve drives heat production to 0 (see Figure 2b), but provides larger electric output.

[1] We report nominal values at an ambient temperature of 15 °C.

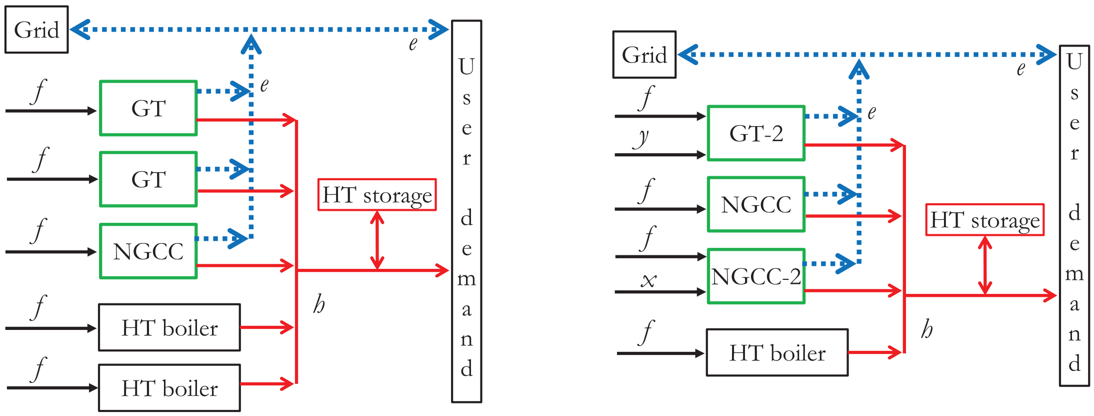


Figure 3: Representation of instances `2-a` and `2-b`.

6.3 Selected results

In Table 1, the type and number of (co)generation units contained in each instance are specified. The performance curves are obtained by fitting experimental or simulated data with quadratic functions.

For scenario 1, we consider two instances. For scenario 2, we consider four different unit configurations. In instances `2-a` and `2-b` (see Figure 3 for a schematic representation), heat demand is relatively low. In instances `2-c` and `2-d`, heat requirements are higher, and more units are necessary to fulfill them. Instances `2-b` and `2-d` include also cogeneration units with two degrees of freedom. In short-term planning, it is common to face instances with a time horizon ranging from a few hours to several weeks. Here we consider a time horizon of two days, with 48 one-hour periods. When taking into account annual economical incentives, we may be even forced to consider one year time horizons (see, e.g., [5]).

For full details on this set of instances in AMPL (`.nl`) and GAMS (`.gms`) format, the reader is referred to [20].

	HP $f \to l$	AB $f \to h$	SOFC $f \to h, e$	NGCC $f \to h, e$	NGCC-2 $f, x \to h, e$	GT $f \to h, e$	GT-2 $f, y \to h, e$	number of units
`1-a`	1	1	1	-	-	-	-	3
`1-b`	2	2	2	-	-	-	-	6
`2-a`	-	2	-	1	-	2	-	5
`2-b`	-	2	-	1	1	-	2	6
`2-c`	-	4	-	4	-	4	-	12
`2-d`	-	4	-	2	1	2	2	11

Table 1: Type of units included in each instance. Input and output variables for each unit are indicated on the second row. For instance, unit NGCC produces heat power h and electric power e from fuel f.

Computational experiments were performed with commercial and open-source global solvers for generic MINLP on an Intel Xeon with E3125@3.30GHz CPUs and 16GB of RAM. Tests were carried out on a single core, with no memory limit, a time limit of 2 hours and a relative optimality gap tolerance of 0.01%. We report results obtained with BARON 14.4 [21], Couenne 0.4 [3] and SCIP 3.1.0 [1], called from GAMS 24.4.

Starting from Formulation (1)-(12), the MINLP model used in the computational experiments is strengthened by adding simple lower and upper bounds to all the decision variables. For concave g_{it}, Constraint (13) can be easily convexified, for example, with a big-M reformulation. When g_{it} is convex and univariate, we also add a valid inequality that provides an approximation of the convex hull of the (non-convex) region defined by the performance constraints[2]. From a practical point of view, scaling is also essential, since the original data often contain values that are several orders of magnitude apart (e.g., generated energy with respect to cost coefficients), leading to numerical difficulties or, sometimes, even infeasible or suboptimal solutions. Note that there has been recent work on strategies to further tighten similar MINLP formulations, such as those based on perspective reformulations (see, e.g., [12, 10]).

Table 2 summarizes a selection of computational results. As usual, the gap is computed as $100\frac{|best-LB|}{\min\{|best|,|LB|\}}$. Observe that, on some instances of the second scenario, where the whole generated power is sold to the grid, it is sometimes possible to find a solution with a negative cost, i.e., a net revenue.

It is worth noting that the difficulty of even small-size instances can vary substantially depending on the structure of the cogeneration system. For example, instances with a few generation units that dominate the others in terms of efficiency and allow to satisfy the whole requests, are significantly easier, which is quite reasonable.

BARON certifies optimality (within the set tolerance) for 3 of the 6 instances, and finds the best known solution for 4 of them. SCIP has a similar behavior. It solves rather easily `1-a` and `2-a`, both within a minute, and achieves good bounds for `2-b`, though it does not close the gap. On instance `1-b`, SCIP is not able to find even a single feasible solution. The instances `2-c` and `2-d` are very challenging for both BARON and SCIP, that are unable to find satisfactory solutions within the time limit.

Couenne is usually not competitive with the other two solvers. On instance `2-b`, it even gives an infeasible solution. We assume this is due to numerical errors or to a bug. Interestingly, on instance `2-d` Couenne provides a good solution that neither BARON nor SCIP are able to find.

	BARON				SCIP				Couenne			
	time	best	LB	gap	time	best	LB	gap	time	best	LB	gap
1-a	**298.3**	**21.48**	**21.48**	**0.01**	**50.5**	**21.48**	**21.48**	**0.00**	7200	25.54	15.88	60.83
1-b	7200	25.65	24.68	3.94	7200	–	24.49	∞	7200	32.58	18.04	80.63
2-a	**502.3**	**24.58**	**24.58**	**0.01**	**58.4**	**24.58**	**24.58**	0.00	7200	24.58	20.81	18.13
2-b	**7155.2**	**-16.27**	**-16.28**	**0.01**	7200	-16.27	-16.32	0.27	*Infeasible solution*			
2-c	7200	86.78	38.28	126.67	7200	–	39.76	∞	7200	–	23.11	∞
2-d	7200	265.86	42.98	518.53	7200	–	44.49	∞	7200	45.75	13.97	227.52

Table 2: Selected results obtained with three MINLP global solvers. We report the best valid upper and lower bounds found within the time limit of 2 hours, and the relative gap %. Certified optima within the tolerance are in bold. The symbol '-' indicates that not even a feasible solution was found. Note that ∞ is used when the gap is meaningless because the two bounds have different signs.

6.4 Concluding remarks

The above selected computational results, for two relatively simple scenarios of a basic version of the short-term operation planning problem with a two-day time horizon, indicate that even small-size instances can be very challenging to solve to optimality. The additional constraints and (binary) variables needed to account for longer time horizons and capture other typical features, such as, for instance, ramp-up/down constraints or minimum and maximum unit uptime requirements, often make the MINLP models corresponding to real-world applications even harder. A natural alternative approach consists in approximating the nonlinear performance curves with piecewise linear functions, see e.g. [4] and [23]. Although the resulting approximate MILP models can generally be solved more efficiently than their MINLP counterparts, the advantages and disadvantages of the two approaches still need to be investigated for complex energy systems involving several interacting cogeneration units with nonlinear performance curves.

[2]For a unit i, the function g_{it} is approximated by connecting in the input-output space (see e.g., Figure 2a) the extreme point $\left(F_{it}^{max}, g_{it}(F_{it}^{max})\right)$ either with the origin $(0, 0)$ or with the point $\left(F_{it}^{min}, g_{it}(F_{it}^{min})\right)$.

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